Quark Orbital Angular Momentum in the MIT Bag Model

Matthias Burkardt and Abdullah Jarrah
Department of Physics, New Mexico State University, Las Cruces, NM 88003-0001, U.S.A.

Using the MIT bag model, we study the contribution from the gluon vector potential due to the spectators to the orbital angular momentum of a quark in the bag model. For $\alpha_s = \mathcal{O}(1)$, this spectator contribution to the quark orbital angular momentum in the gauge-covariant Ji decomposition is of the same order as the non gauge-covariant quark orbital angular momentum and its magnitude is larger for d than for u quarks and negative for both.

PACS numbers:

I. INTRODUCTION

Since the famous EMC experiments revealed that only a small fraction of the nucleon spin is due to quark spins [1], there has been a great interest in 'solving the spin puzzle', i.e. in decomposing the nucleon spin into contributions from quark/gluon spin and orbital degrees of freedom. In this effort, the Ji decomposition [2]

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q^z + J_g^z \tag{1}$$

appears to be very useful, as not only the quark spin contributions Δq but also the quark total angular momenta $J_q \equiv \frac{1}{2}\Delta q + L_q^z$ (and by subtracting the spin piece also the the quark orbital angular momenta L_q^z) entering this decomposition can be accessed experimentally, through generalized parton distributions (GPDs). The terms in (1) are defined as expectation values of the corresponding terms in the angular momentum tensor

$$M^{0xy} = \sum_q \frac{1}{2} q^\dagger \Sigma^z q + \sum_q q^\dagger \left(\vec{r}\times i\vec{D}\right)^z q + \left[\vec{r}\times\left(\vec{E}\times\vec{B}\right)\right]^z \tag{2}$$

in a nucleon state with zero momentum. Here $i\vec{D} = i\vec{\partial} - g\vec{A}$ is the gauge-covariant derivative. The main advantages of this decomposition are that each term can be expressed as the expectation value of a manifestly gauge invariant local operator and that the quark total angular momentum $J_q^z = \frac{1}{2}\Delta q + L_q^z$ can be related to generalized parton distributions (GPDs) [2] and is thus accessible in deeply virtual Compton scattering and deeply virtual meson production and can also be calculated in lattice gauge theory.

Recent lattice calculations of GPDs surprised in several ways [3]. First, the light quark orbital angular momentum (OAM) is consistent with $L_u \approx -L_d$, i.e. $L_u + L_d \approx 0$, which would imply that $J_g \approx \frac{1}{2}\cdot 0.7$ represents the largest piece in the nucleon spin decomposition. Secondly, $L_u \approx -0.15$ and $L_u \approx +0.15$ in these calculations, i.e. the opposite signs from what one would expect from many quark models with relativistic effects, as we will also illustrate in the following section. While the inclusion of still-omitted disconnected diagrams may change the sum $L_u + L_d$, it does not affect the difference $L_u - L_d$. In Ref. [4], it was pointed out that evolution from a quark model scale of few hundred MeV to the lattice scale of few GeV could possibly account for the difference. However, due to the presence of interactions through the vector potential in the gauge covariant derivative L_q^z does not have a parton interpretation, which complicates its physical interpretation.

Jaffe and Manohar have proposed an alternative decomposition of the nucleon spin, which does have a partonic interpretation [5]

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q \mathcal{L}_q^z + \frac{1}{2}\Delta G + \mathcal{L}_g^z, \tag{3}$$

and whose terms are defined as matrix elements of the corresponding terms in the $+12$ component of the angular momentum tensor

$$M^{+12} = \frac{1}{2}\sum_q q_+^\dagger \gamma_5 q_+ + \sum_q q_+^\dagger \left(\vec{r}\times i\vec{\partial}\right)^z q_+ + \varepsilon^{+-ij}\mathrm{Tr}F^{+i}A^j + 2\mathrm{Tr}F^{+j}\left(\vec{r}\times i\vec{\partial}\right)^z A^j. \tag{4}$$

The first and third term in (3),(4) are the 'intrinsic' contributions (no factor of $\vec{r}\times$) to the nucleon's angular momentum $J^z = +\frac{1}{2}$ and have a physical interpretation as quark and gluon spin respectively, while the second and fourth term

can be identified with the quark/gluon OAM. Here $q_+ \equiv \frac{1}{2}\gamma^-\gamma^+ q$ is the dynamical component of the quark field operators, and light-cone gauge $A^+ \equiv A^0 + A^z = 0$ is implied. The residual gauge invariance can be fixed by imposing anti-periodic boundary conditions $\mathbf{A}_\perp(\mathbf{x}_\perp, \infty^-) = -\mathbf{A}_\perp(\mathbf{x}_\perp, -\infty^-)$ on the transverse components of the vector potential.

Only the Δq are common to both decompositions. While for a nucleon at rest the difference in the Dirac structure between L_q^z and $\mathcal{L}_q^z$ plays no role [7], the appearance of the gluon vector potential in the operator defining L_q^z implies that in general $\mathcal{L}_q^z \neq L_q^z$. Indeed, in Ref. [7] their difference was illustrated in the context of an electron in QED to order α. This lowest order correction to the electron orbital angular momentum from its own vector potential, can be directly translated into a QCD correction to the orbital angular momentum of a quark, resulting in [7]

$$L_q^z = \mathcal{L}_q^z - \frac{\alpha_s}{3\pi} \tag{5}$$

for a quark state with $J^z = +\frac{1}{2}$ (and $\vec{p} = 0$). However, in addition to this contribution to $q^\dagger \left(\vec{r} \times g\vec{A}\right) q$ from the $\vec{A}$ field 'caused' by its own current, the vector potential can also have been caused by the spectators, or may be due to intrinsic glue. In this note, we use the bag model to make an estimate for such 'spectator effects', by examining the contribution of the vector potential arising from spectator currents to $L_q^z - \mathcal{L}_q^z$ in perturbation theory to $\mathcal{O}(\alpha_s)$.

II. ORBITAL ANGULAR MOMENTUM IN THE MIT BAG MODEL

In the bag model [6], the Dirac wave function for a quark state with $j^z = s$, is of the form

$$\psi_s = \mathcal{N} \begin{pmatrix} j_0(kr)\chi_s \\ ij_1(kr)\hat{\vec{r}} \cdot \vec{\sigma}\chi_s \end{pmatrix} \tag{6}$$

where χ_s is a two-component Pauli spinor, $k = E = \frac{2.0428}{R}$, with R being the bag radius, and $\mathcal{N} = \mathcal{N}_s$ is a normalization constant. For simplicity, we take the quarks to be massless.

Evaluating the orbital angular momentum is straightforward in the absence of QCD corrections. For a state with $s = +\frac{1}{2}$ one finds from the lower component of the Dirac wave function

$$\mathcal{L}_q^z = |\mathcal{N}|^2 \, \frac{8\pi}{3} \int_0^R drr^2 j_1^2(kR) = 0.1735 \tag{7}$$

One can easily understand the sign of (7): since the quark spin $s^z = \pm\frac{1}{2}$ and its orbital angular momentum must add up to $j^z = \frac{1}{2}$ for each wave function component, the $\hat{z}$-component of the quark orbital angular momentum can only be $+1$ or 0, i.e. $0 \leq \mathcal{L}_q^z \leq 1$. For a quark state with $j_q^z = -\frac{1}{2}$, one obtains of course the opposite sign for $\mathcal{L}_q^z$.

Using standard $SU(6)$ wave functions,

$$\begin{aligned} |p\uparrow\rangle = \frac{1}{\sqrt{18}} \; &\{2\,|u\uparrow u\uparrow d\downarrow\rangle + 2\,|u\uparrow d\downarrow u\uparrow\rangle + 2\,|d\downarrow u\uparrow u\uparrow\rangle - |u\uparrow u\downarrow d\uparrow\rangle - |u\downarrow u\uparrow d\uparrow\rangle \\ &- |u\uparrow d\uparrow u\downarrow\rangle - |u\downarrow d\uparrow u\uparrow\rangle - |d\uparrow u\uparrow u\downarrow\rangle - |d\uparrow u\downarrow u\uparrow\rangle\}, \end{aligned} \tag{8}$$

this result gets multiplied by $\frac{4}{3}$ for u quarks in a proton and by $-\frac{1}{3}$ for d quarks in a proton, yielding

$$\begin{aligned} \mathcal{L}_{u/p}^z &= \frac{4}{3} \cdot 0.1735 = 0.2313 \\ \mathcal{L}_{d/p}^z &= -\frac{1}{3} \cdot 0.1735 = -0.0578, \end{aligned} \tag{9}$$

with signs opposite to that of the lattice calculation [3]. A similar pattern for the signs as in Eq. (9) is observed in all quark models where the orbital angular momentum arises as a relativistic effect from the lower Dirac component. Similarly, phenomenological models for single-spin asymmetries have the same pattern of signs for quark angular momenta as the bag model (9) and thus also differ from the lattice QCD results [3]. Although present lattice calculations still suffer from uncontrolled systematic errors due to the omission of operator insertions into disconnected quark loops, this does not affect the isovector combination $\mathcal{L}_{u/p}^z - \mathcal{L}_{d/p}^z$, which is positive in the model calculations but negative in lattice QCD. Q^2 evolution has been proposed as a possible explanation to account for this apparent discrepancy [4, 9], but in order to accomplish quantitative consistency, perturbative evolution equations need to be used very deep in the nonperturbative regime.

Since quark models have, to lowest order in α_s, no vector potential, it makes perhaps more sense to identify the quark OAM from these models with $\mathcal{L}_q^z$ rather than with the GPD-based L_q^z. In Ref. [7] the difference $\Delta_e^z \equiv \mathcal{L}_e^z - L_e^z$ was calculated to order α for a single electron in QED and the result then also applied to a single quark in QCD. However, in QCD quarks are never alone and the question arises regarding the effects from 'spectator currents' on the orbital angular momentum of each quark.

In order to address this issue, we will in the following focus on estimating $\mathcal{O}(\alpha_s)$ corrections to the difference

$$\Delta_q^z \equiv L_q^z - \mathcal{L}_q^z = \langle q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\rangle. \tag{10}$$

The vector potential in (10) is calculated from the spectator currents, which are obtained by taking matrix elements in the corresponding ground state bag model wave functions. The vector potential resulting from these static currents is obtained by solving

$$\vec{\nabla}^2 \vec{A}^a(\vec{r}) = -\vec{j}^a(\vec{r}) = -\sum_{s'} g\psi_{s'}^\dagger(\vec{r})\vec{\alpha}\frac{\lambda^a}{2}\psi_{s'}(\vec{r}) \tag{11}$$

for each color component a and where the summation is over the spectators (here we pick the gauge $\vec{\nabla}\cdot\vec{A}^a = 0$, but to $\mathcal{O}(\alpha_s)$ the result is actually gauge invariant as we will explain below).

The contribution from a spectator with $j^z = s'$ to $\langle q(\vec{r}\times\vec{A})^z q\rangle$ thus reads

$$\Delta_{s'}^z = -\frac{2}{3}\frac{g^2}{4\pi}\int d^3r d^3r' \psi_s^\dagger(\vec{r})\psi_s(\vec{r})\psi_{s'}^\dagger(\vec{r}')\frac{(\vec{r}\times\vec{\alpha})^z}{|\vec{r}-\vec{r}'|}\psi_{s'}(\vec{r}') \tag{12}$$

where the factor $-\frac{2}{3}$ arises from the color part of the matrix element. Note that $\vec{\Delta}_{s'}$ is independent of the angular momentum $j^z = s$ of the 'active quark', since $\psi_{\frac{1}{2}}^\dagger(\vec{r})\psi_{\frac{1}{2}}(\vec{r}) = \psi_{-\frac{1}{2}}^\dagger(\vec{r})\psi_{-\frac{1}{2}}(\vec{r})$. However, it depends on the spin of the spectator since the orientation of the vector potential entering (10) depends on the latter. For example, for $s' = +\frac{1}{2}$, one finds

$$\psi_{s'}^\dagger(\vec{r}')(\vec{r}\times\vec{\alpha})^z\psi_{s'}(\vec{r}') = |\mathcal{N}|^2 j_0(kr')j_1(kr')2\frac{xx'+yy'}{r'} \tag{13}$$

and hence

$$\Delta_{+\frac{1}{2}}^z = -\frac{2}{3}\alpha_s I_R \tag{14}$$

with

$$I_R = |\mathcal{N}|^4\int_{r<R} d^3r\int_{r'<R} d^3r' \left[j_0^2(kr)+j_1^2(kr)\right]2\frac{xx'+yy'}{|\vec{r}-\vec{r}'|}\frac{j_0(kr')j_1(kr')}{r'}. \tag{15}$$

The angular integration can be easily done using the expansion $\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^\infty \frac{4\pi}{2l+1}\sum_{m=-l}^{l}\frac{r_<^l}{r_>^{l+1}}Y_l^m(\Omega)Y_l^{m*}(\Omega')$ and noting that only $l=1$ contributes in (15), yielding

$$I_R = \frac{4}{9}(4\pi)^2|\mathcal{N}|^4\int_0^R dr\int_0^R dr' r'^2 r^3\left[j_0^2(kr)+j_1^2(kr)\right]\frac{r_<}{r_>^2}j_0(kr')j_1(kr') = 0.1165 \tag{16}$$

which does not depend on the bag radius. For $s' = -\frac{1}{2}$ the result is identical with a negative sign.

If the active quark has s alligned with that of the proton the two spectators must have opposite s' and their contribution to $\vec{\Delta}$ cancels, i.e. $\vec{\Delta}$ is nonzero only in those wave function components where the active quark has $s = -\frac{1}{2}$, in which case both spectators have $s' = +\frac{1}{2}$. As a result, $\Delta_{q/p}^z$ is equal to twice $\Delta_{+\frac{1}{2}}^z$ times the probability to find that quark flavor with $s = -\frac{1}{2}$ (which is $\frac{1}{3}$ for $q = u$ and $\frac{2}{3}$ for $q = d$, and hence

$$\begin{aligned}\Delta_{u/p}^z &= \frac{2}{3}\Delta_{+\frac{1}{2}}^z = -\frac{4}{9}\alpha_s I_R = -0.052\alpha_s \\ \Delta_{d/p}^z &= \frac{4}{3}\Delta_{+\frac{1}{2}}^z = -\frac{8}{9}\alpha_s I_R = -0.104\alpha_s\end{aligned} \tag{17}$$

which is the main result of this paper.

For simplicity, we evaluated the vector potential in the $\vec{\nabla}\cdot\vec{A}=0$ gauge. However, our perturbative result is gauge invariant (to $\mathcal{O}(\alpha_s)$) at least in the subclass of all gauges where all color components are treated (globally) SU(3)-symmetrically. I such gauges, matrix elements of operators of the type $q^\dagger\lambda^a\Gamma q A^a$, where Γ is some Dirac matrix, and A^a is calculated to $\mathcal{O}(\alpha_s)$, are proportional to the matrix elements of the corresponding abelian operators. Therefore it is sufficient to establish gauge invariance of $q^\dagger\vec{r}\times\vec{A}q$ for abelian fields. The key observation is that the MIT bag-model wave functions contain no correlations between the positions of the quarks. Therefore, after eliminating the color in this calculation and introducing abelian currents, $\Delta^z_{s'}$ factorizes into the density of the active quark $\psi_s^\dagger(\vec{r})\psi_s(\vec{r})$ times $(\vec{r}\times\vec{A})_z = rA_\phi$. Writing the volume integral $\int d^3r$ in cylindrical coordinates, one can isolate the only ϕ-dependent term $r\int_0^{2\pi}d\phi A_\phi = \oint d\vec{r}\cdot\vec{A}(\vec{r})$ as a closed loop integral with fixed r and z. The closed loop integral is gauge invariant (its numerical value represents the color-magnetic flux through a circle with radius r) and so is the volume integral in which it enters.

In addition to (1) and (3), other angular momentum decompositions have been proposed in the literature (see for example Refs. [8, 9]). Since we used Coulomb gauge, $\vec{A}=\vec{A}_{phys}$, where $\vec{A}_{phys}$ is the transverse part of the vector potential as defined in Refs. [8, 9]. In the decomposition proposed in Ref. [8], the quark orbital angular momentum involves only the longitudinal part of the vector potential, which is zero in our calculation, and which, in the bag model, as well as many other quark models, vanishes also in other gauges to $\mathcal{O}(\alpha_s)$ as discussed above. Therefore, within our approximations, the quark orbital angular momentum as introduced in Ref. [8] would not be affected by spectator effects and would be given by Eq. (9). In contradistinction, the quark orbital angular momentum in Ref. [9] is identical to that of [2] and therefore would receive the same contribution from the vector potential as L^z_q.

III. DISCUSSION

We found that the effect from the vector potential caused by spectators on $\Delta^z_{q/p}\equiv L^z_q-\mathcal{L}^z_q$ is negative for both $q=u,d$. This is consistent with the sign that one may have intuitively expected: since $\vec{A}$ is obtained using the Biot-Savard equation $\vec{A}(\vec{r})=\frac{1}{4\pi}\int d^3r'\frac{\vec{j}(\vec{r})}{|\vec{r}-\vec{r}'|}$, one would expect that $\vec{A}$ and $\vec{j}$ have the same orientation. Hence $\vec{r}\times\vec{A}$ should have the same sign as the magnetization density $\vec{r}\times\vec{j}$. In the sum over all spectators, quarks with $j_z=+\frac{1}{2}$ dominate and thus the magnetization density of the spectators has the same sign as the nucleon spin. Taking into consideration the negative color factor which reflects the fact that the interaction between the quarks is attractive, and the fact that only spectators with s' contribute, one arrives at the above negative sign.

The difference $\Delta^z_{u/p}-\Delta^z_{d/p}=0.052\alpha_s$ is positive. This implies that the vector potential from the spectators adds a negative contribution to $\mathcal{L}^z_u-\mathcal{L}^z_d$ when compared to $L^z_u-L^z_d$. Since $L^z_u-L^z_d$ calculated in lattice QCD is already negative, subtracting our result for $\Delta^z_{u/p}-\Delta^z_{d/p}$ would make it even more negative, and therefore this does not help in understanding the puzzling lattice results.

The numerical values of $\Delta^z_{q/p}$ are of the same order of magnitude as the corresponding difference for a free quark [7]. Whether our result is overall numerically significant depends on the numerical value of α_s — a matter of debate in the context of the bag model. For example, in order to generate the experimentally measured $N-\Delta$ mass splittings from perturbative color-hyperfine interactions in the bag model, $\alpha_s=\mathcal{O}(1)$ is required — in which case the $\Delta^z_{q/p}$ would be large enough that $L_{d/p}$ would become positive, but still not large enough to render $L_{u/p}$ negative. Of course, such a large value of α_s is inconsistent with the use of perturbation theory, and may only reflect the need for correlations between quark wave functions, which are not present in the MIT bag model. However, nonperturbative correlations that enhance the hyperfine splitting, may also enhance Δ^z_q. Since both Δ^z_u and Δ^z_d are negative, their difference is small which seems to indicate that spectator contributions to the isovector combination L_u-L_d are small, but may not be small in the case of the isoscalar combination L_u+L_d.

Acknowledgements: This work was supported by the DOE under grant number DE-FG03-95ER40965.

[1] J. Ashman et al (EMC), Phys. Lett. B **206**, 364 (1988); Nucl. Phys. B **328**, 1 (1989).
[2] X. Ji, Phys. Rev. Lett. **78**, 610 (1997).
[3] Ph. Hägler *et al.* (LHPC Collaboration), Phys. Rev. D **77**, 094502 (2008).
[4] A.W. Thomas, and F. Myhrer, Phys. Lett. B **663**, 302 (2008); A.W. Thomas, Phys. Rev. Lett. **101**, 102003 (2008).
[5] R.L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).
[6] A. Chodos et al., Phys. Rev. D**9**, 3471 (1974); T.A. DeGrand et al., *ibid.* **12** 2060 (1975).
[7] M. Burkardt and H. BC, Phys. Rev. D**79**, 071501 (2009).
[8] X.S. Chen et al., Phys. Rev. Lett. **100**, 232002 (2008); **103**, 062001 (2009).

[9] M. Wakamatsu, Phys. Rev. D **81**, 114010 (2010); Eur. Phys. J. A **44**, 297 (2010).